

# Flow of granular matter in a silo with multiple exit orifices: Jamming to mixing

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We investigate the mixing characteristics of dry granular material while draining down a silo with multiple exit orifices. The mixing in the silo, which otherwise consists of noninteracting stagnant and flow regions, is observed to improve significantly when the flow through specific orifices is stopped intermittently. This momentary stoppage of flow through the orifice is either controlled manually or is chosen by the system itself when the orifice width is small enough to cause spontaneous jamming and unjamming. We observe that the overall mixing behavior shows a systematic dependence on the frequency of closing and opening of specific orifices. In particular, the silo configuration employing random jamming and unjamming of any of the orifices shows early evidence of chaotic mixing. When operated in a multipass mode, the system exhibits a practical and efficient way of mixing particles.

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## I. INTRODUCTION

Flow of granular media through a hopper or silo has a ubiquitous presence in several industrial applications and has been investigated over a long time. The flow consists of particle drainage under the influence of gravity and comprises an accelerating section close to the exit orifice and a slow plug region high above the orifice. Various modeling approaches have been used to describe the entire flow behavior, viz., the kinematic model [1], the void model [2], the spot model [3], the frictional-cosserat model [4] and many others. The output flow rate from the silo is known to follow the famous Beverloo correlation over a wide range of system parameters [5]. The system is, however, more known, or quite notorious, for its uncanny ability to jam suddenly due to the formation of highly stable arches at the exit orifice [6–8] which can cause many problems in industrial operations.

Typically, the particles in a silo are densely packed, and when they flow, the motion is quite slow in the transverse direction as compared with the downward direction. Additionally, the system is comprised of stagnant zones, particularly in a rectangular bottom silo, which does not show significant particle motion throughout the silo drainage. All these characteristics add up to result in little or almost non-existent mixing behavior, as evidenced through the advection dominated particle dynamics in the downward flow direction compared with transverse diffusion [9].

In this work, we try to ascertain whether a silo can be used for efficient granular matter mixing. We seek motivation from our recent work [10] wherein we observed the effect of nonlocal flow behavior on the jamming characteristics in a hopper with multiple exit orifices. We showed that the fluctuations emanating due to flow from one orifice of a silo can cause another jammed orifice located as far as 30 particle diameters to spontaneously

unjam and flow. When the two orifices are very close to each other (less than 3 particle diameters), the jamming of orifices can be prevented due to mutually interacting arches over both orifices [11]. We extend this concept of jamming and unjamming to demonstrate that, in a silo with multiple exit orifices, the particles above a jammed orifice can pass through the other adjacent flowing orifice, resulting in substantial cross flow. The result is the enhanced mixing behavior observed in the system. We show this behavior using two different ways of creating the desired cross flow in the system, viz., (i) closing (or jamming) and opening (or unjamming) of an orifice in a systematic, controlled manner and (ii) allowing the orifices in the silo to jam and unjam randomly depending on their interorifice distance and orifice width. It is to be noted that this enhanced mixing behavior using both schemes is enabled through flow re-arrangements created noninvasively and hitherto unobserved in a silo system. Such enhanced mixing behavior will be of immense importance to several applications wherein the silo acts as a feeder to a process requiring uniformly mixed material or is a part of a larger integrated system and requires efficient mixing throughout the height as in a nuclear pebble bed reactor. We explore this enhanced mixing behavior in detail by using discrete element method (DEM) simulations of soft particles. The particle positions obtained throughout the course of simulation are used to evaluate the spatial and temporal dependence of the mixing properties, measured as degree of mixing.

## II. METHODOLOGY

The DEM simulations are carried out using the Large Atomic-Molecular Massively Parallel Simulator (LAMMPS) developed at Sandia National laboratories [12, 13]. The simulation employs Hookean force between two contacting particles described in detail elsewhere [14]. All the simulation parameters are the same as used in the systematic study of silo flows carried out previously [14], except for a higher normal elastic constant

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( $k_n = 2 \times 10^6 mg/d$ ) which corresponds to a more stiff particle. The interparticle friction coefficient ( $\mu$ ) is varied from 0.2 to 0.8 with no qualitative difference between the results. Here, we report the results obtained for  $\mu = 0.5$ .

The simulation geometry (see Fig. 1) consists of a rectangular, flat-bottomed, silo of height  $H$ , width  $L$ , and thickness  $3d$ , where  $d \pm 0.15d$  is the particle diameter with a uniform size distribution. The side and bottom walls are created by freezing the largest sized particles so that their translational and angular velocities are kept zero throughout the simulation run. The silo has five orifices, each of width  $D$  and separated by the inter-orifice distance  $w$ . Periodic boundaries are used in the  $y$  direction (in and out of the paper), which represents an infinitely long silo in  $y$  direction. The silo is filled by using the sedimentation method as suggested previously [15] in which a dilute packing of nonoverlapping particles is created in a simulation box and allowed to settle under the influence of gravity. The simulation is run for a significant time so that the kinetic energy per particle is less than  $10^{-8} mgd$  resulting in a quiescent packing of height  $H$  in the silo. All the particles in the silo are the same type and size, but colored differently to create five different vertical bands, each centered above one of the five orifices.

The silo system is operated in two ways, viz., (i) single-pass and (ii) multipass. In a single-pass system, the particles traverse down the entire height of the silo only once before exiting through the orifice located at the bottom. The height ( $H$ ) of each column of (colored) particles is maintained constant at  $160d$  by inserting new particles of same color at the free surface equal in number to those which have exited from the system per unit time. The free surface remains nearly flat during the entire simulation run. This mode of operation represents an infinitely tall silo or a silo with a finite height continuously receiving a fresh feed of material. In contrast, in a multi-pass system every particle traverses down the entire silo height several times during the simulation run. The same particles are re-inserted at the free surface and at the same horizontal position from where they exited the system. The average fill height ( $H$ ) during the simulation run remains constant at  $80d$  while the free surface can exhibit significant slope, as discussed later. This mode of operation represents a batch system with the particles subjected to a particular flow mechanism repeatedly. A vertical cascade of silos, wherein the material exiting from one silo would enter at the free surface of another silo located exactly below it, would represent such a batch system.

The flow through the silo system is controlled using two protocols. In one scenario, the second and fourth orifice from either sidewall are opened and closed alternately while keeping the remaining three (first, third, and fifth) orifices open throughout. The orifice (either second or fourth) is kept open, while keeping the fourth or second closed, respectively, for a time ( $\Delta t$ ) within which the mean distance drop of the particles in the silo is ap-

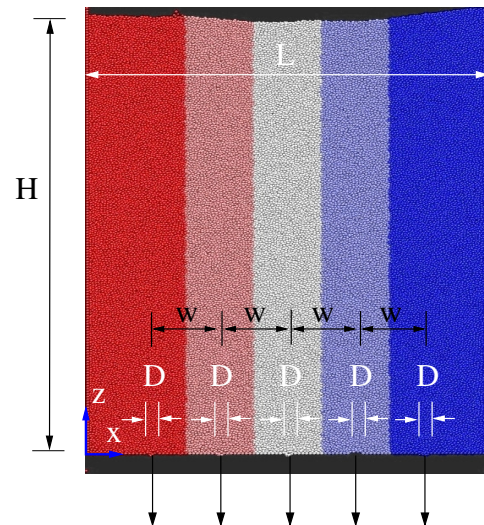


FIG. 1. (Color online) Sample snapshot of the initial silo before the flow ensues. All the particles in the system are of same size and polydispersity. The color code is employed to visualize and identify particles over different orifices and in different regions.

proximately  $T = \langle v_z \rangle \Delta t / d$ , where  $\langle v_z \rangle$  is the absolute magnitude of the downward velocity of particles averaged across the silo width. The mixing characteristics are determined for various values of  $T$ . The simulations for each value of  $T$  are continued until the total number  $n$  of instances of closing and opening of orifices reach fourteen (both second and fourth orifices opened and closed seven times each). The total run-time  $t$  of the simulations is, thus, different for each value of  $T$ . Here time  $t$  is measured in terms of unit  $\tau = \sqrt{d/g}$ , which is the natural timescale of simulations and  $g$  is the acceleration due to gravity. To stop the flow through an orifice, the particles in a region  $3d \times 5d$  just above the orifice are suddenly frozen. Similarly, to re-initiate the flow, these frozen particles are allowed to fall freely under gravity. In an experimental system, this is equivalent to a rapid closing and opening of the orifice valve, respectively, to stop and restart the flow. The width ( $D$ ) of each orifice is  $3d$  which ensures a constant flow rate from the orifice as long as the orifice is kept open. The interorifice distance ( $w$ ), i.e., the center-to-center distance between adjacent orifices, is kept constant throughout at  $15d$ . The overall silo width ( $L$ ), excluding the side-wall particles, is  $105d$  wherein each extreme orifice is placed  $22.5d$  away from the nearest sidewall. In the subsequent sections, we refer to this protocol as the “controlled mechanism”.

In another scenario, the width of all the orifices is reduced to  $2d$  while the interorifice distance is maintained almost the same ( $w \approx 15d$ ). This change in the system causes a dramatic change in the flow behavior. The smaller orifice width can cause jamming of flow, only to be unjammed due to fluctuations transferred from one of the other, albeit intermittently flowing, adjacent ori-

fices [10]. The probability of this unjamming depends on the distance between two adjacent orifices and the orifice width. For a given orifice width, the closer the two orifices, the more probable is unjamming, while this probability is very small when the orifices are far apart. In the latter case, the fluctuations cannot traverse that long to unjam a jammed orifice. The combination of  $w$  and  $D$  used here allows for this random jamming and unjamming to persist without complete stoppage of flow and ensuring flow through at least two orifices at any point of time. The resulting effect is that the material above the jammed orifice can cross over to the adjacent region, leading to mixing. However, the value of  $T$  is now variable throughout the course of simulation, which is chosen randomly by the system on its own. Unlike the controlled-mechanism case where only a specific orifice can close or open, in this case one or more orifices can randomly jam or unjam. It should be noted that different combinations of  $D$  and  $w$  are possible, albeit within a very limited range, to initiate this random jamming-and unjamming-induced flow. We, however, focus here on the fixed  $D$  and  $w$  mentioned above to simply provide a flavor of the mixing dynamics arising out of this flow scheme. The width ( $L$ ) of the silo, excluding the side-wall particles, in this case is  $74.5d$  wherein the extreme orifices are placed  $7.5d$  away from the nearest sidewall. We refer to this protocol as the “random mechanism” in subsequent sections.

To characterize mixing in the system, a column of particles above individual orifices are colored differently (see Fig. 1). A horizontal box of height ( $h = 4d$ ) and width ( $l = 18d$ ) and depth  $3d$  is centered exactly at a distance  $4d$  above each orifice [see Fig. 2(a) and 2(b)]. The cumulative particle fraction  $\langle\phi\rangle$  for the entire silo at any time ( $t$ ) is defined as

$$\langle\phi\rangle = \frac{\sum \phi_i}{n_o} = \frac{\sum N_i(t)/N_i(0)}{n_o}, \quad (1)$$

where  $i$  denotes the region above each orifice,  $\phi_i$  is the fraction corresponding to box ( $h \times l \times 3d$ ) above each orifice, and  $n_o$  is the number of orifices. The summation is carried over the values from all five boxes located, respectively, over five orifices. Initially, each box ( $h \times l \times 3d$ ) contains particles of only one color and the number is  $N_i(0)$ . The number of particles of that color in the same box at different times ( $t$ ) is  $N_i(t)$ . The cumulative particle fraction, thus, measures the fraction of particular colored particles in a box preserved over time ( $t$ ). The degree of mixing in the system is defined as

$$M = \frac{1 - \langle\phi\rangle}{1 - 1/n_o}. \quad (2)$$

For five orifices ( $n_o = 5$ ) in the system, the value of  $\langle\phi\rangle$  can vary from 1 for “no mixing” to 0.2 for “complete mixing”. Correspondingly, the value of  $M$  varies from 0 (no mixing) to 1 (complete mixing). Measurements of  $M$  over time provides steady state evolution of mixing

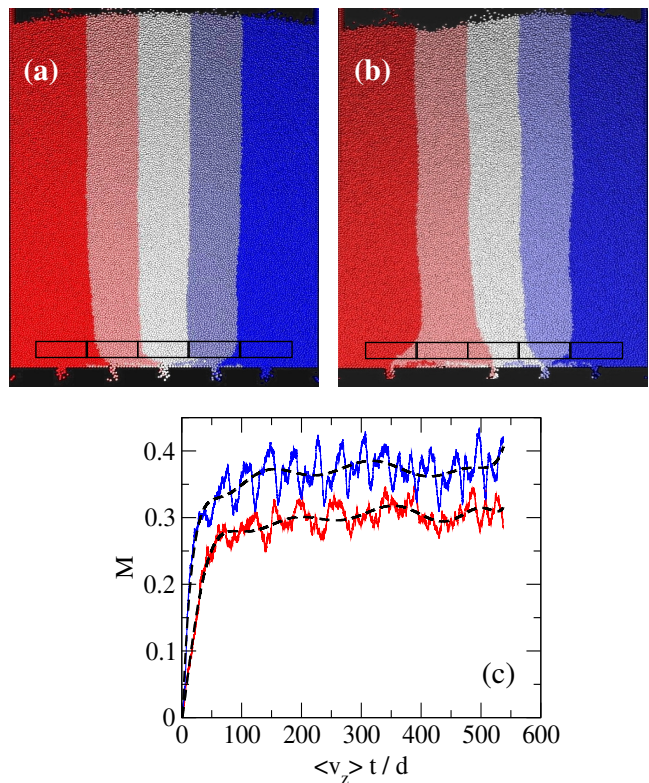


FIG. 2. (Color online) Mixing patterns and degree of mixing for various orifice configurations. Snapshots obtained for (a) all orifices open ( $T = 0.0$ ), (b) second and fourth orifices opened and closed alternately, keeping  $T = 38.4$  at  $\langle v_z \rangle t/d = 400$ . Black boxes at the bottom represents the region in which  $M$  is calculated. (c) Variation in the degree of mixing ( $M$ ) with time for two cases: red (lower curve) denotes the configuration in panel (a), and blue (upper curve) denotes the configuration in panel (b). The dashed black lines show a tenth-degree-polynomial fit.

behavior. Similarly, measurements in the same sized box but located at different heights provides the spatial variation of mixing.

### III. RESULTS

#### A. Single pass system

In this section, we discuss the results obtained using different flow-controlling methods in the system through which every particle traverses only once. We first discuss the mixing behavior for manually controlled flow through orifices. Figure 2(b) shows a snapshot during the flow obtained for  $T = 38.4$ , which is the largest value considered. Also shown in Fig. 2(a) is the case for  $T = 0$  (i.e., all orifices open at all times). The mixing for intermediate values of  $T$  is discussed later. More details about the pattern evolution can be observed in the movies uploaded as supplementary material for different cases. The steady state

evolution of the degree of mixing is shown in Fig. 2(c). For both the cases the simulations are continued for a time duration  $t$  so that the value of  $M$  reaches a near constant value and the total mean downward distance traveled ( $\langle v_z \rangle t/d$ ) is the same in both the cases. To obtain an estimate of the steady state value, a tenth-order polynomial is fit to the entire data set [dashed black lines shown in Fig. 2(c)]. The steady state value ( $M_s$ ) is then obtained from the average of the polynomial-fit values over a time domain within which its standard deviation is less than 1%. In Fig. 2(c), this time domain corresponds to  $200 \leq \langle v_z \rangle t/d \leq 600$  for both the cases. The images shown in Figs. 2(a) and 2(b) are obtained at time instant  $\langle v_z \rangle t/d = 400$ .

When all orifices are kept open at all times, the system represents, to a certain extent, five single-orifice silos of smaller widths operating in parallel. We find particles flowing within all regions of the silo, thus, eliminating the presence of any stagnant regions, particularly near the bottom corners, typically observed in a single-orifice flat-bottomed silo. It is observed that the particles from one column (of a particular color) flow through the orifice located exactly below the adjacent column [of another color; see Fig. 2(a)]. The reason for this is the interaction between the spatial regions above each orifice. Note that, for a single-orifice silo, a convergent-flow region exists just above the orifice which extends vertically upwards up to a height approximately equal to the width of the silo while simultaneously also extending horizontally up to a distance approximately equal to half the silo width. The typical downward velocity profile across the width at any height in the convergent section has a Gaussian shape [1, 3]. It is, thus, quite natural for the flow fields above each orifice, in a multiple exit orifice silo, to interact with each other in a nonlinear manner. The particles on or near the interface of the two adjacent columns will, thus, have an equal probability of going through either of the two adjacent orifices, leading to cross flow and mixing of particles with  $M_s \approx 0.3$  [see Fig. 2(c)]. The profile after reaching steady state shows sustained oscillations about the value of  $M_s$  obtained from the polynomial fit. The amplitude and period of these oscillations are governed by the time-dependent inherent dynamics of the coupling between two adjacent flow fields.

For the controlled mechanism with  $T = 38.4$ , more cross flow is observed, which also extends to larger heights above the orifice [see Fig. 2(b)]. The degree of mixing evolves to a steady state value of 0.375 [shown as (upper) blue curve in Fig. 2(c)] which is higher than for the case  $T = 0.0$ . Here as well, the value of  $M$ , after reaching steady state, continues to oscillate. The oscillations in this case arise due to sequential closing and opening of orifices, causing particles from a particular column to flow through the orifice located below the adjacent column. The particle fraction values in the region above second or fourth orifice then oscillate depending on the opened or closed state. This affects the values of the cumulative particle fraction and eventually the value of

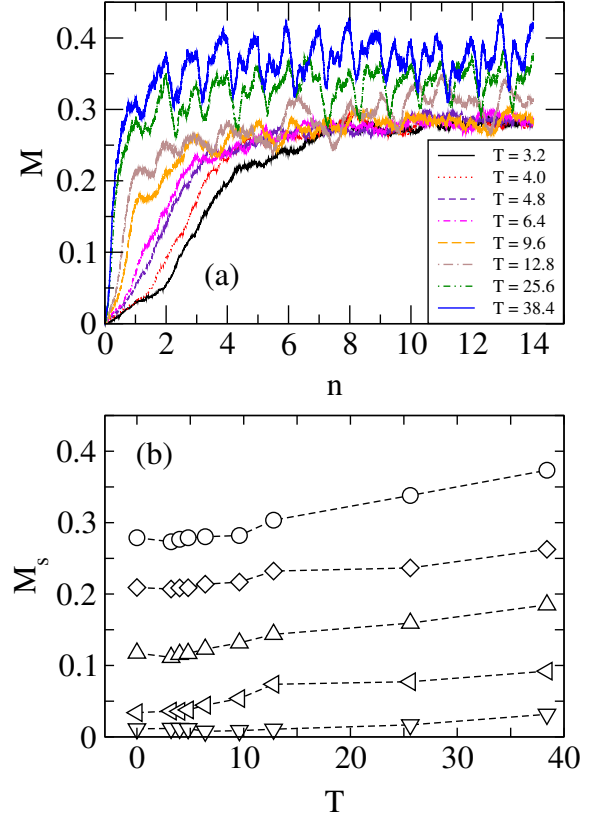


FIG. 3. (Color online) (a) Degree of mixing ( $M$ ) plotted against  $n$ . Increased  $M$  for increasing values of  $T$  at any  $n$ . (b) Final steady state value ( $M_s$ ) obtained from the values of the polynomial fit (not shown) plotted against  $T$ . Symbols denote values obtained at different heights above the orifice:  $\circ z = 4d$ ,  $\diamond z = 6d$ ,  $\triangle z = 8d$ ,  $\triangleleft z = 30d$ ,  $\nabla z = 110d$ .

$M$ . The period of oscillations in the profile of  $M$  is approximately equal to  $T$  and the amplitude is governed by the number of particles cross flowing within time  $T$ . Figure 2(b) shows a snapshot at one particular instant for this flow scheme. However, different cross-flow patterns can be observed during the flow; viz., “white” particles (center column) moving through the second and third orifices and the “dark red” (leftmost column) and “dark blue” (rightmost column) moving, respectively, through the second and fourth orifice. The material collected from each orifice, on an average, now contains uniformly mixed particles obtained from three different regions.

The effect of variation of  $T$  on the steady state evolution of mixing is shown in Fig. 3(a). For each value of  $T$ , the number of instances ( $n$ ) of opening and closing of orifices is kept constant at fourteen. The total run-time of simulation ( $t$ ), then, increases with increasing  $T$  to achieve the steady state mixing. For higher values of  $T$ , fewer instances ( $n$ ) of closing and opening the orifices are necessary to reach steady state. Furthermore, the values of  $M$  increase with increase in  $T$  for any value of  $n$ . For larger values of  $T$ , the time avail-



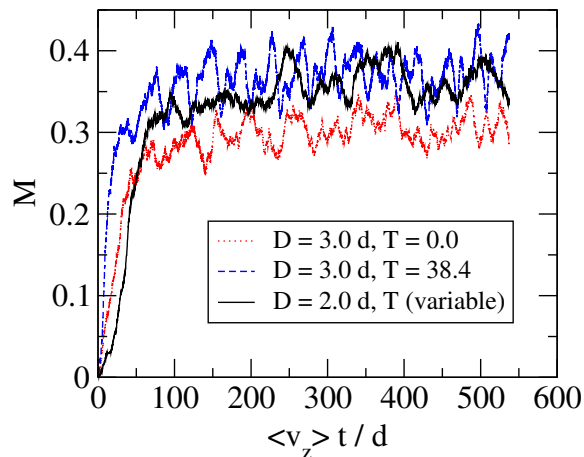


FIG. 4. (Color online) Evolution of mixing with time for different orifice diameter and  $T$ . Results for  $D = 2.0d$  are obtained by allowing system to choose the  $T$  which adjusts to the spontaneous jamming and unjamming occurring at different orifices.

able for the cross flow across the regions above different orifices is greater, leading to higher mixing, hence higher  $M$ . The amplitude of the oscillations of the steady-state continue to decrease with decreasing  $T$ . For smaller  $T$ , the duration of cross flow is quite small (hence smaller period) and consequently smaller amplitude (i.e., lesser particles cross flowing during a particular sequence). For very small values of  $T$  ( $\approx 0.0$ ), the system asymptotically approaches the flow and mixing behavior observed in Fig. 2(a).

The final steady state value of  $M$  obtained from the profiles in Fig. 3(a) are shown in Fig. 3(b) (symbols shown as open circles). The steady state value for all cases is obtained by using a tenth-degree-polynomial fit, as described previously. The value of  $M_s$  shows maximum change (by about 15%) for  $T > 10$ . For lower values of  $T$ , the degree of mixing is nearly the same as obtained when all orifices are kept open ( $T = 0$ ). Figure 3(b) also shows the values of  $M_s$  measured at different heights above the orifice for varying values of  $T$ . The same value of  $n$  is needed to achieve the steady state at all heights, i.e., the steady state is achieved everywhere simultaneously (profiles not shown). For any value of  $T$ , the degree of mixing decreases with increasing height above the orifice. The dependence on  $T$  is qualitatively the same for all heights up to  $30d$ , above which the material moves like a plug and seems unaffected by the controlled-mechanism protocol employed. The value of  $M_s$  remains close to 0.0, irrespective of the value of  $T$  employed.

Next, we compare the mixing behavior for different flowing protocols, viz., controlled and random mechanisms. In the former, only the second and fourth orifices can close and open, and that too with a definite period,  $T$ . In the latter case, one or more orifices can jam or unjam any number of times, leading to variable  $T$ . The

evolution of mixing for this latter case along with that for the controlled mechanism with  $T = 38.4$  and the base case of  $T = 0$  is shown in Fig. 4. The total run-time  $t$  in simulations is adjusted so that the mean distance traveled by particles remains the same in all three cases. The value of  $M_s$  for the random mechanism is quite close to that obtained for  $T = 38.4$ , although it is achieved a bit slowly. In the initial period, the degree of mixing is even lower than that for  $T = 0.0$ . The periodicity of the oscillations are not defined clearly due to the highly variable  $T$  in this protocol. This protocol, as mentioned earlier, has limited range within which the interorifice distance and the orifice width can be varied. Increasing the orifice width ( $D$ ) to  $2.2d$  creates flow similar to that for  $T = 0.0$ , i.e., there are hardly any instances of flow jamming and the flow never stops from any of the orifices. While for widths ( $D$ ) below  $2d$  all five orifices tend to jam together, thus stopping the flow completely, which can be restarted only by external intervention (remove a particle from the arch or vibrate the system). Over the simulation run-time ( $t$ ), the total number of particle e-flux is the least for the random mechanism while it is the most for  $T = 0.0$ .

## B. Multipass System

Here, we present results for a multipass system, i.e., the situation wherein every particle, initially filled in the silo, is made to traverse down the entire silo height multiple times. The results are presented only for the case of random mechanism using  $D = 2d$ ,  $w = 15d$ , and  $H = 80d$ . The results for the controlled mechanism are qualitatively similar, as evident from Fig. 4, and hence are not presented. Certain quantitative differences, though, are discussed at appropriate places.

The mixing patterns are shown in the snapshots in Fig. 5(a)-5(d) for the initial state and after different flow cycles ( $N = 1, 4, 10$ ). One flow cycle represents every particle in the silo traversing the entire height ( $H$ ) once and would approximately correspond to  $v_z \Delta t / d = 80d$ . The movie showing the entire simulation run is provided as supplementary material. From the images of the initial state and that after the 10th cycle, it seems as if significant mixing has taken place across the width of the silo. This is confirmed by the value of the degree of mixing  $M$  calculated in the region covered by the black boxes of same size and located at same height as used in a single-pass system. The final steady state value of  $M$  is quite high, around 0.75 (see Fig. 6), but still lower than 1.0 which represents complete mixing. It seems that complete mixing will be achievable after an infinite number of cycles, as evident from the very slow rate of change in the value of  $M$  after eight cycles.

The overall mixing behavior in the multi-pass system can arise out of three different mechanisms. The first can be due to the impact of the particles hitting the free surface and then bouncing horizontally in the nearby re-

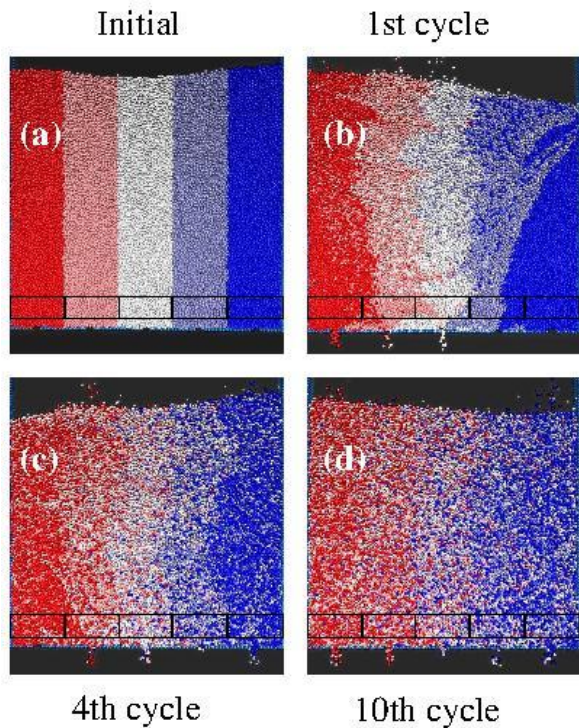


FIG. 5. (Color online) Mixing patterns in a silo ( $D = 2d$ ,  $w = 15d$ ) operated as a multipass system. Opening and closing of any orifice is chosen based on random jamming and unjamming events. Black boxes at the bottom represent the region in which the particle fraction is calculated.

gions. This does not seem to be the case, as observed from the particle trajectories (not shown over here). The particles simply fall onto the free surface and start moving downward showing very little lateral movement due to impact. The second mechanism is due to slope formation at the free surface [see Fig. 5(b)] leading to an avalanche of particles, thereby transferring the particles to nearby regions, thus causing horizontal spreading. This is evident from the horn of “light-blue” particles (second column from right wall) spreading up to the silo wall, as seen clearly in Fig. 5(b). The creation of local slope is due to the certain columns remaining stationary due to jammed orifice below them and all the particles exiting from other orifices and piling up at the corresponding free surface above. The third mechanism is due to the particles in a region above a certain orifice flowing out from the nearby orifice due to jamming of the orifice below it and then re-entering the silo in the zone from where it exited instead of where it started in the previous cycle. This is the same mechanism responsible for the profiles of  $M$  shown in Fig. 4 but is applied every cycle. The relative dominance of these two mechanisms is difficult to ascertain quantitatively. The second mechanism is expected to be dominant when orifices are jammed for an extended period of time causing the material to pile up for avalanching at the free surface. In the scenario where

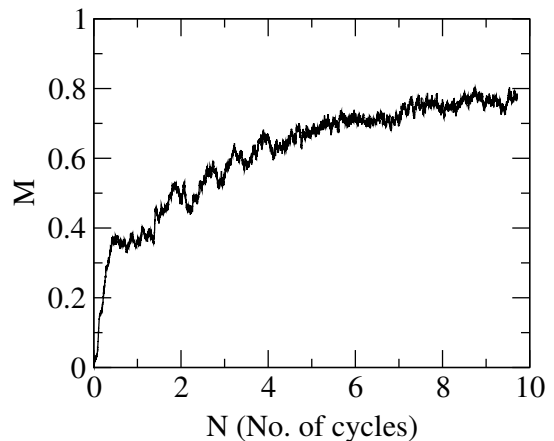


FIG. 6. Change in the degree of mixing with flow cycles. The maximum value of mixing achievable when all colored particles are distributed uniformly across the horizontal distance is 1.

there is rapid jamming and unjamming, the third mechanism is expected to dominate.

The repeated application of the second and third mechanisms per cycle eventually causes a particle to drift significantly from its original horizontal position at the start of the simulation run. Even a few of the particles from the regions in the vicinity of the wall manage to reach the opposite side wall of the silo. However, this number is very restricted as the bulk of the particles in the extreme columns do not show significant horizontal drift even after ten cycles, thus preventing complete mixing. To some extent, this is also due to the limitation enforced on the particles in the first and fifth columns to move only in one direction, the other being blocked by the sidewall. The final mixed state of the silo shown in Fig. 5(d) is observed to be independent of the height above the orifice, i.e., the same horizontal variation is observed at all heights.

If the controlled mechanism (with highest  $T$ ) is employed instead of the random mechanism in a multipass mode, the degree of mixing achieved at all times (cycles) is about 10% lower. In the latter case, every orifice closes at least once with few others closing multiple times and for any duration and, furthermore there are certain instances of two orifices being closed simultaneously. The overall result is to enable particles from all regions of the silo to undergo the mixing process, which results in a more homogeneous mixture. While for the controlled mechanism, only one (second or fourth) orifice is closed at any point of time and that, too, for a fixed time period. Thus, only certain specific regions (mostly the three central columns) of the silo get involved in mixing, resulting in less homogeneous mixing. It is obvious that the controlled mechanism, if modified to close and open all orifices for a varied time, would result in improved mixing of the same order, if not better, as for the case of the random mechanism. However, the scheme employ-

ing random mechanism seems more easily realizable in practice.

We would like to note that the phenomena of horizontal shifting of the particle positions at every flow cycle has striking similarities with those encountered in a quasi-two-dimensional horizontal rotating cylinder [16–18], which show chaos due to changing streamline positions. In a rotating cylinder, this streamline shifting is achieved due to flowing layer thickness fluctuations induced by varying rotation rate or using cylinders of non-circular crosssection. Here, this shifting is achieved by the inherent randomness in the opening and closing of the orifices. It is not clear at the moment if the eventual mixing behavior is a result of possible chaos achieved in the system. This would require a more extensive study using a model which can exhibit flow discontinuities in a jamming and unjamming scenario. Nevertheless, it is quite interesting to know about the existence of possible chaos in a silo system which arises out of system randomness.

#### IV. SUMMARY

To summarize, our study shows that the mixing behavior of particles draining down a silo can be enhanced by using multiple orifices and clever manipulation of the choice of orifices which can close and open frequently. The closing of an orifice leads to the crossflow of particles between different regions, thus improving mixing. More importantly, the overall scheme employed eliminates the stagnation zones in the system while ensuring a nearly uniform particle fraction across the silo width,

a feature missing in the currently used single-orifice silos. The desired level of mixing can either be obtained by controlled opening and closing of specific orifices or allowing the system to choose on its own using the inherent random jamming and unjamming behavior. Different schemes employed provide a realistically usable system in practice: operating the silo with a smaller height in a single or multipass mode by repeated recirculation of the particles emanating from the exit orifices. Operating over longer times would ensure a highly uniform mixture of particles across the entire silo, a feature which is not that easy to achieve even in a more familiar rotating cylinder system. The results obtained by subjecting particles to repeated horizontal crossflow show promise for achieving a chaos in the system arising out of the inherent system randomness. While the overall results show promise for dry particulate systems, their applicability to a more complicated and practically relevant cohesive granular system remains to be explored further. In that case, the orifice size and interorifice distance may have to be varied, perhaps, to account for a large effective particle size due to formation of clusters of individual cohesive particles. More interesting would be to study the mixing behavior of mixtures of particles of varying size and/or density which would need varying orifice sizes and interorifice distances in the same system.

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